

# A quick tour of regular (evenly spaced) temperaments:

- Pythagorean
  - Equal
  - Meantone

Eleven 5ths of one consistent size:  
either pure, or some controlled impure size

- The one leftover diminished 6<sup>th</sup> is:
- Smaller than the 5ths (in Pythagorean),
  - Larger than the 5ths (in meantone systems), or
    - The same size as the 5ths (in equal)

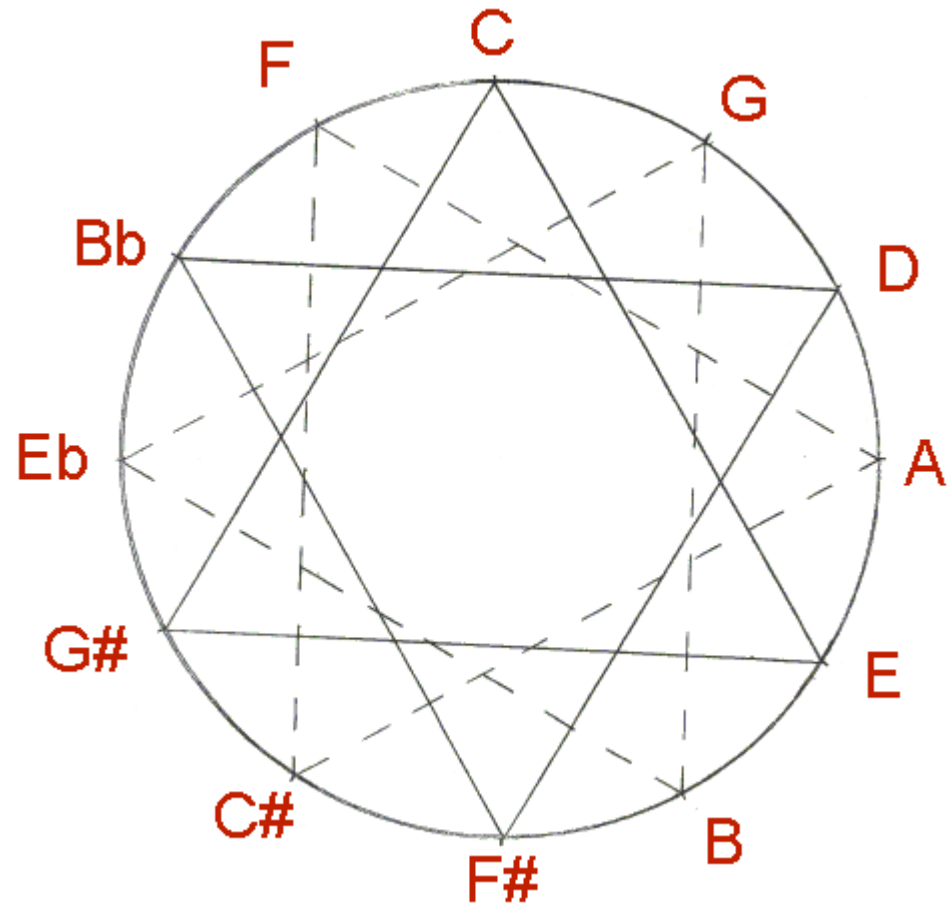
## “Pythagorean” tuning:

Eleven pure 5ths

One leftover diminished 6<sup>th</sup>  
where the circle cannot  
close: flats do not meet  
sharps, and they are not  
interchangeable

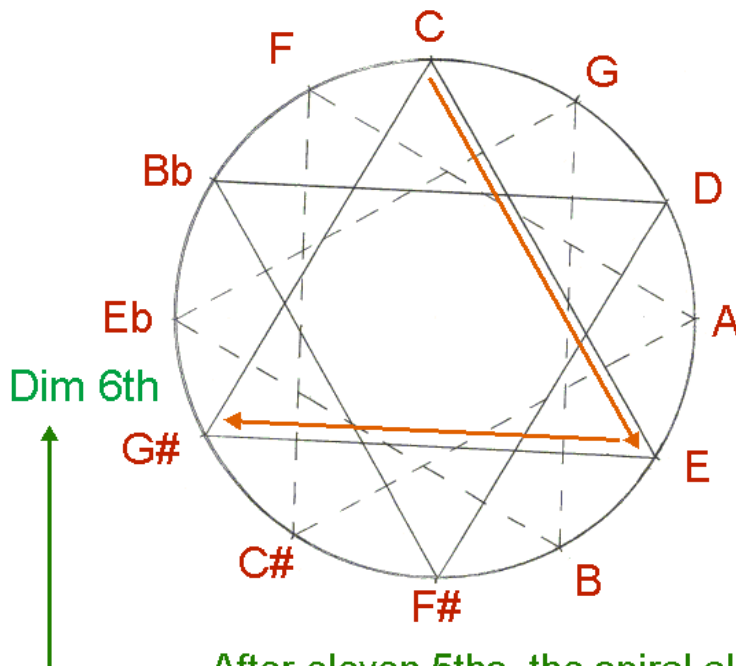
The diminished 6<sup>th</sup> is too  
narrow by one  
“Pythagorean comma” (a  
measurement of the error)

Major 3rds that are too  
wide to sound harmonious



# Pythagorean tuning

Eleven pure 5ths.



Four 5ths (less a "syntonic comma") build a major 3rd.

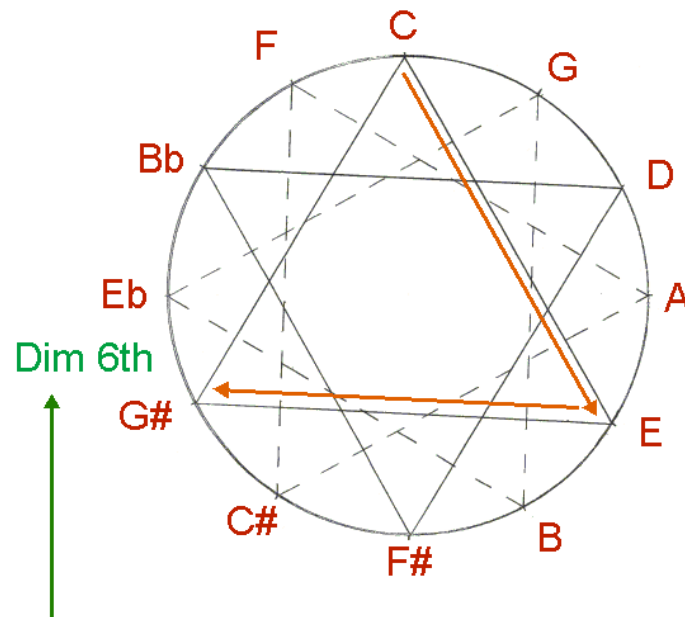
Two major 3rds plus a diminished 4th build an octave.

After eleven 5ths, the spiral almost meets itself.  
On 12-note keyboards we have a diminished 6th at the overlap point.

$E_b$  and the missing  $D\#$  are different from each other by one "Pythagorean comma". Similarly,  $G\#$  and  $A_b$  are one PC apart.

# Equal temperament

Make all eleven 5ths slightly narrow by the same small amount.



Unfortunately, the slight narrowing of the 5ths in equal temperament is not enough to make the major 3rds sound very good.

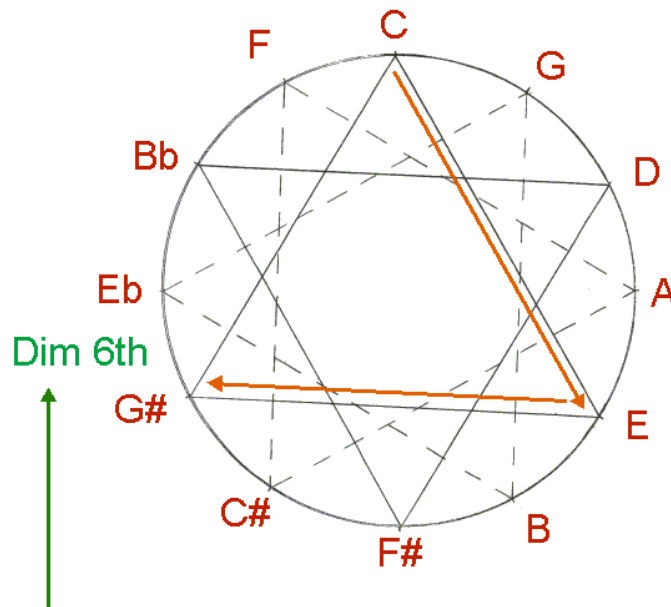
They are still too wide by  $\frac{7}{11}$  of a "syntonic comma".

We can choose some different but consistent ("regular") size of 5th, instead of pure 5ths. The diminished 6th's gap ends with a different size.

In equal temperament, each 5th is narrowed by  $\frac{1}{12}$  PC, so the gap at the diminished 6th is 0. It sounds exactly like the other (narrowed) 5ths.

# Regular temperaments (“meantone”)

Make all eleven 5ths narrower than they are in equal temperament.



In "meantone" temperaments, the 5ths are made narrower than in equal temperament.

The goal is to make the major 3rds better, at the expense of the 5ths.

In the meantone temperaments, the diminished 6th makes a wide gap.

The D# is much lower than the Eb. If we choose to install an Eb, it sounds wrong to play that note as if it were D#...and vice versa.

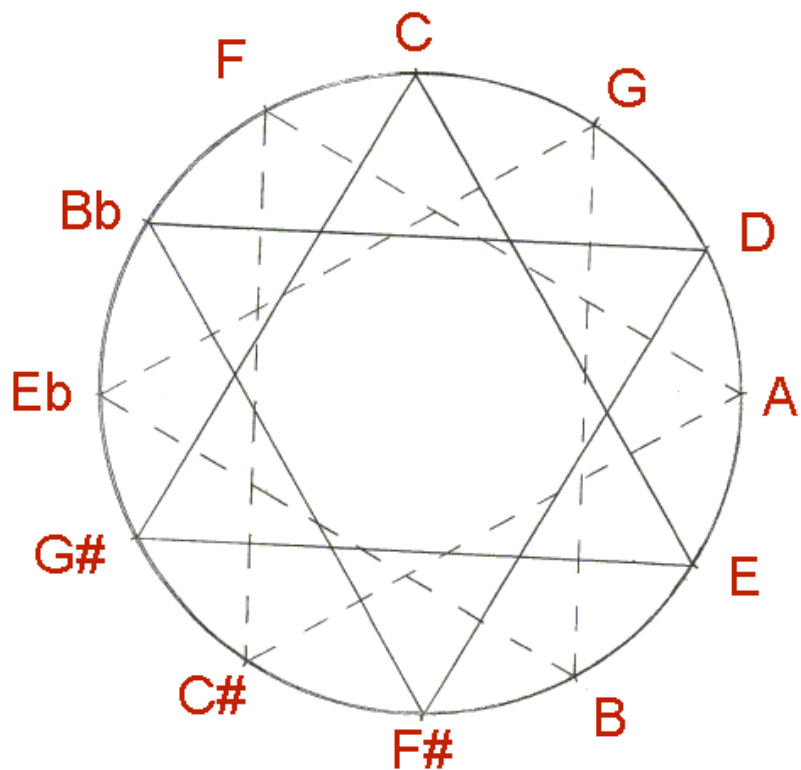
In tonal music, we really have a “spiral of 5ths”,  
not a “circle of 5ths” ...

There are approximately 28 to 31 differently named  
notes used in “common practice” music :

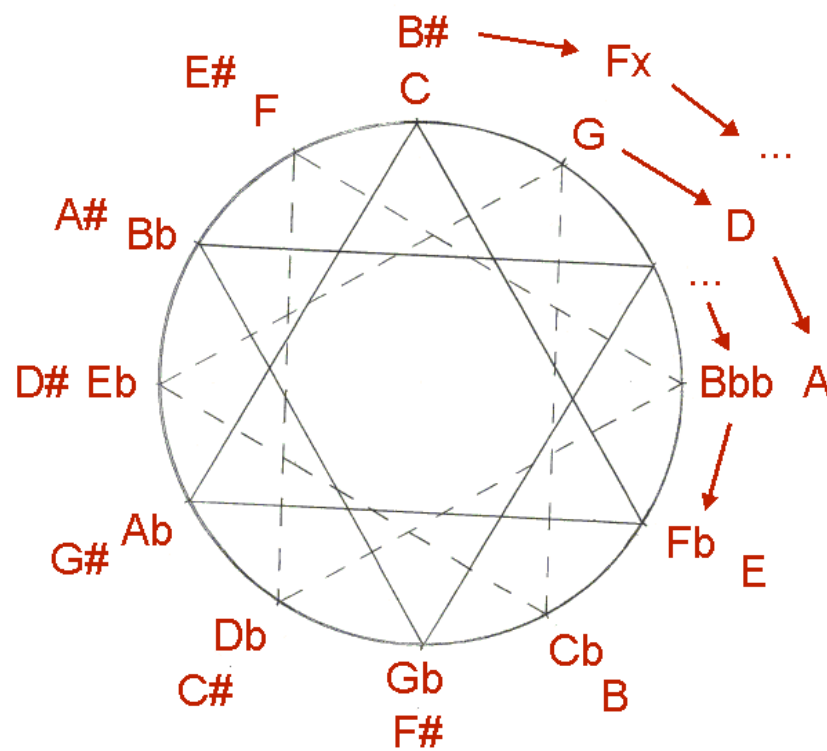
..., [Gbb, Dbb, Abb,] Ebb, Bbb,  
Fb, Cb, Gb, Db, Ab, Eb, Bb,  
F, C, G, D, A, E, B,  
F#, C#, G#, D#, A#, E#, B#,  
Fx, Cx, Gx, Dx, Ax, ...

# Which subset of notes do we need to install into our keyboard instruments?

## 12 most common note names



## Spiral of 5ths

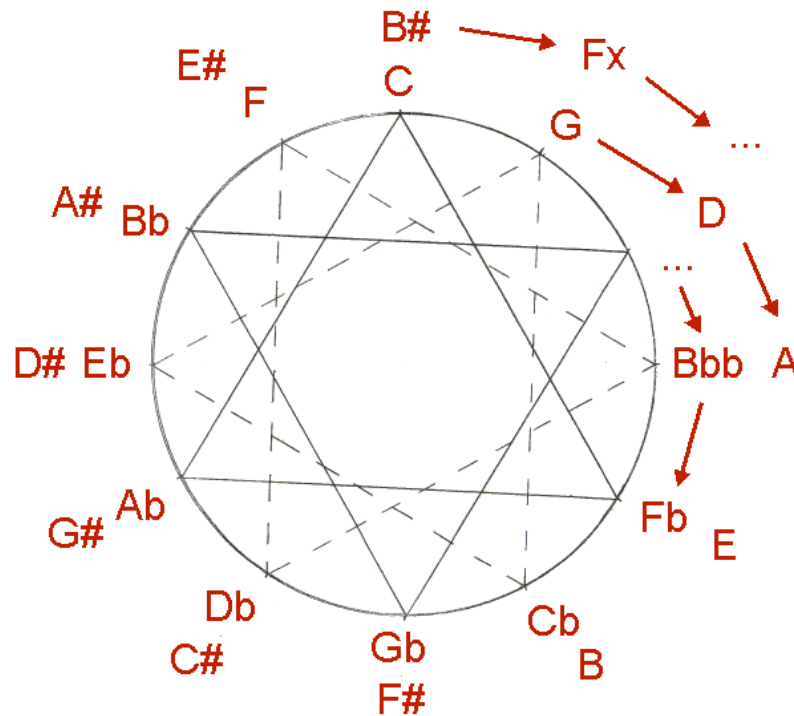


The 12 most commonly-used notes in Western music :

A, Bb, B, C,  
C#, D, Eb, E,  
F, F#, G, G#,  
A....



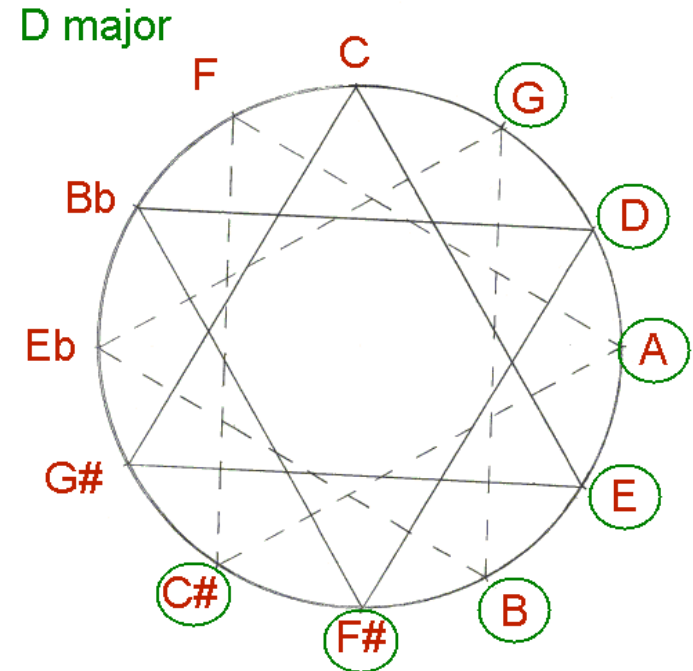
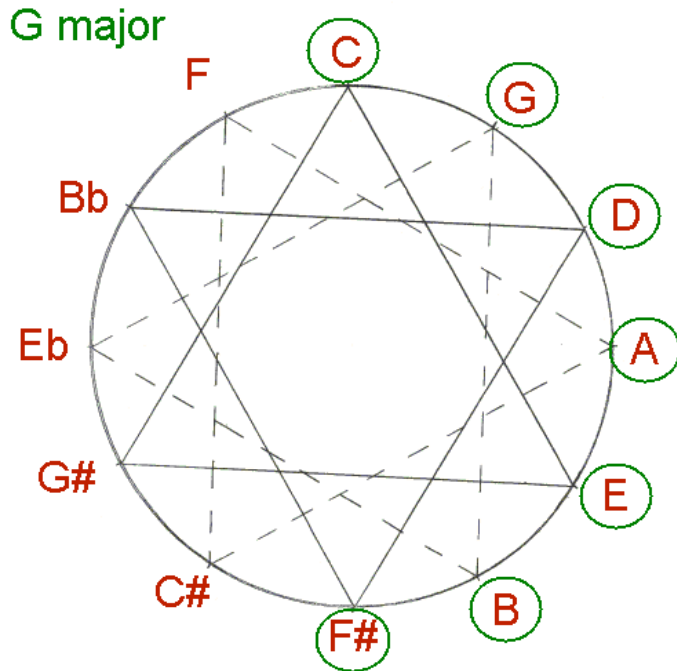
# That spiral of 5ths again



Keep rotating the Fa-Ut-Sol-Re-La-Mi-Ti sequence as much as you like, starting anywhere: get a different major scale each time. It is endless in both directions....

Reviewing modulation: the old Ut becomes the new Fa, or vice versa.

**The whole Fa-Ut-Sol-Re-La-Mi-Ti sequence simply rotates to a new position.**

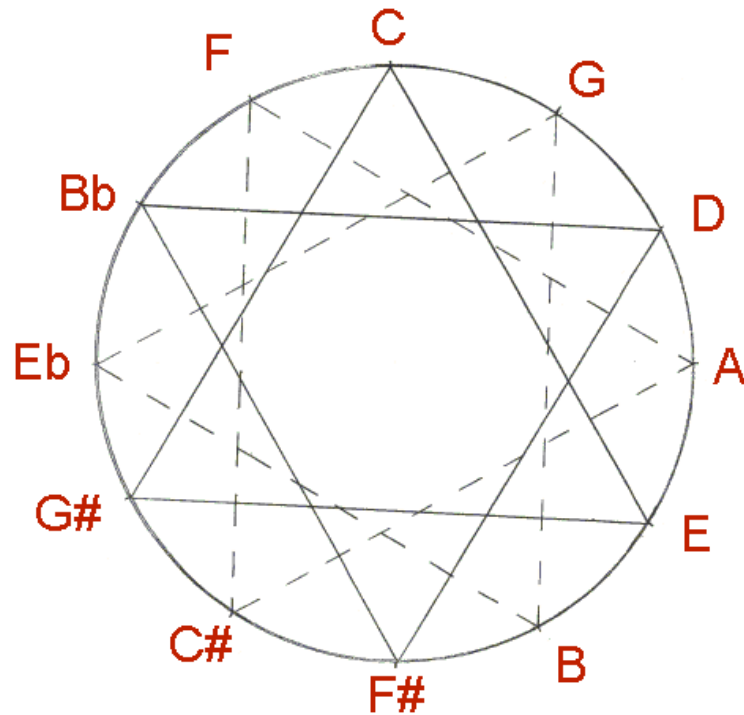


If we keep rotating it clockwise, modulating sharpward, we will get a D#, A#, E#, B#, etc. If we rotate it the other way, modulating flatward, we will get Ab, Db, Gb, Cb, etc.

**“Meantone”  
temperaments tend  
to favor these 12  
notes, and let all the  
other notes be wrong.**

The tuner puts the most common notes into positions where they work well, both harmonically and melodically.

Because the exotic notes such as D#, A#, Ab, and Db won't be played in the composition, they don't need to be available at all (or in tune).



**The Fa-Ut-Sol-Re-La-Mi-Ti cycle works for only six major scales: Bb, F, C, G, D, and A. The other major scales all have misspelled notes in them.**

Eb major fails because Fa (Ab) is missing. E major fails because Ti (D#) is missing. B major fails because both Mi and Ti are missing....

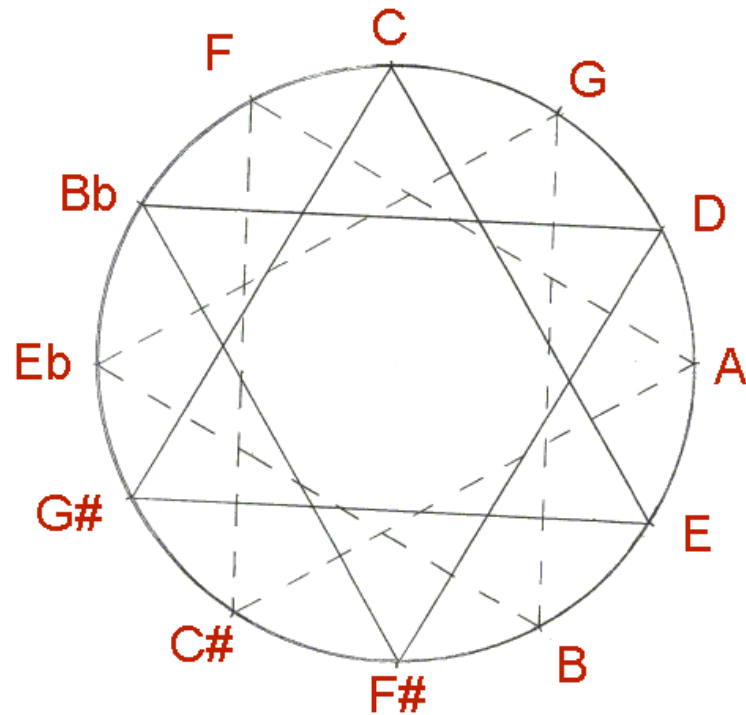
## That's a problem for tonal music.

G major music frequently needs a D#. There isn't one.

D major music often needs both D# and A#.

Bb major music often needs an Ab, and sometimes also a Db. They aren't available.

A minor music (with zero sharps or flats in the signature) often needs a D#. E minor certainly needs a D#, and usually also an A#....



G minor music often needs an Ab, and C minor certainly needs Ab. F minor lacks Ab and Db....

**So...there is trouble playing in anything but F major, C major, and D minor! As soon as the music starts to modulate, we quickly run out of correctly-spelled and correctly-tuned notes.**

# Demonstration of some regular “meantone” features

- Excellent harmony...when the notes are spelled correctly
- Interesting difference between two different types of semitones (half steps): chromatic and diatonic
- Interesting difference between major 3rds and the much wider diminished 4ths: especially at dominant trills in minor keys
- Wolf diminished 6<sup>th</sup> at the gap between sharps and flats
- Disastrous dissonances when the notes are not spelled correctly